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136. "Suppose a planked floor with thin visible seams between the planks. Let there be a thin straight rod not so long as the breadth of the planks. This rod, being tossed up at hazard, will either fall quite clear of the seams or will lie across one seam. Prove that in the long run the fraction of the whole number of trials in which a seam is intersected, will be the fraction which twice the length of the rod is to the circumference of the circle having the breadth of the plank for its diameter."

SOLUTION BY WILLIAM HOOVER, BELLEFONTAINE, OHIO.

Let $2a$ = width of planks, $2b$ = length of rod, φ = the angle the rod makes with a perpendicular to the seams, and let x = the distance of the center of the rod from the nearest seam. The rod will cross a seam for all values of x from 0 to $b\cos\varphi$; hence the chance of crossing the seam is $4\varphi\div 2\pi$, and as the chance that the center of the rod will take the particular position at the distance x from the seam is $dx\div a$, the required probability will evidently be

$$\frac{2}{\pi a}\int \varphi dx = \frac{2b}{\pi a}\int_0^{\frac{1}{2}\pi} \varphi \sin\varphi \, d\varphi = \frac{2b}{\pi a}.$$

Note, by Christine Ladd, Union Springs, N. Y. — The relation between the sides and diagonals of the contra parallelogram given in problem 118 is a particular case of the relation between the two values of the side of a triangle when the remaining sides and the angle opposite one of them are given. If c be the side required and B the angle given, we have

$$c = a \cos B \pm b \cos a,$$
$$cc' = a^2 \cos^2 B - b^2 \cos^2 a = a^2 - b^2.$$

SOLUTION OF PROBLEMS IN NO. SIX, VOL. III.

Solutions of problems in No. 6, Vol. III, have been received as follows: From Marcus Baker, 137; Henry Gunder, 137 and 139; H. Heaton, 139, 140 and answer to query; Artemas Martin, 139 and 140; W. L. Marcy, 137 and 140; Prof. J. Scheffer, 138; E. B. Seitz, 137 and 140; Prof. D. Trowbridge, 140; R. J. Adcock, answer to query.

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137. "A point D, is given in position between two lines which make a given angle at A. Find the position of a given line, BC, drawn through D, and intersecting the two lines in the points B and C."

SOLUTION BY MARCUS BAKER, U. S. COAST SURVEY, WASHINGTON, D. C.

Call the given line *BC*, l, the angle made by the lines given in position, α, and the angle made by the given line l, with one of the lines given in position, θ; then, calling the co-ordinates of the given point a and b, we have (see ANALYST, Vol. II, p. 192)

$$\frac{l}{b\sin\alpha} = \operatorname{cosec}\theta + \frac{a}{b}\operatorname{cosec}(\alpha+\theta)$$

as the equation of the line.

For brevity put $b\sin\alpha = m$, $b\cos\alpha = n$ and $\operatorname{cosec}\theta = x$; whence, since

$$\operatorname{cosec}(\alpha+\theta) = \frac{\operatorname{cosec}\theta}{\sin\alpha\surd(\operatorname{cosec}^2\theta - 1) + \cos\alpha}$$

we have

$$\frac{l}{m} = x + \frac{ax}{m\surd(x^2-1)+n} = \frac{(a+n)x + mx\surd(x^2-1)}{n+m\surd(x^2-1)},$$

and hence $\quad (lm - mx)\surd(x^2 - 1) = (a+n)x - ln.$

Squaring and developing we get, after reduction,

$$x^4 - 2lx^3 + \left[l^2 - \frac{(a+n)^2}{m^2} - 1\right]x^2 + 2l\left[1 + \frac{(a+n)n}{m^2}\right]x - \frac{l^2}{m^2}\left(m^2+n^2\right) = 0.$$

If the given lines are at right angles to each other, i. e., if $\alpha = 90°$ we have $m = b$ and $n = 0$, whence our equation becomes for rectangular co-ordinates

$$x^4 - 2lx^3 + [l^2 - (a^2 \div b^2) - 1]x^2 + 2lx - l^2 = 0.$$

Since the equation is of the 4th degree there are in general four positions of the line which fulfill the conditions of the problem.

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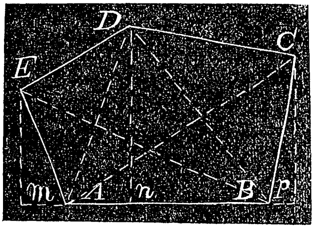
138. "In a pentagon, *ABCDE*, the triangles $ABE = a$, $ADE = b$, $CDE = c$, $BCD = d$, $ABC = e$ are known; to find the area F of the pentagon."

SOLUTION BY PROF. J. SCHEFFER., COLLEGE OF ST. JAMES, MD.

Denoting the co-ordinates *Am* by x, *Em* by y, *AB* by z, *Ap* by t, *Cp* by u, *An* by v, *Dn* by w, we find the equations:



(1) $yz = 2a$,
(2) $zu = 2e$,
(3) $zw = 2(F - b - d)$,
(4) $xw + vy = 2b$,
(5) $tw - uv = 2(F - b - e)$,
(6) $ty + xu = 2(F - c - e)$.

Multiplying (1) by (5), we obtain

(7) $$yztw - yzuv = 4(Fa - ab - ae).$$

Multiplying (2) by (4), we get

(8) $$xzuw + yzuv = 4be.$$

Adding (7) and (8), we get

(9) $$xzuw + yztw = 4(Fa - ab - ae + be).$$

Multiplying (3) by (6), we get

(10) $$xzuw + yztw = 4(F - b - d)(F - c - e).$$

Equating the right-hand members of (9) and (10), we have

$$4(Fa - ab - ae + be) = 4(F - b - d)(F - c - e),$$

whence $F^2 - (a+b+c+d+e)F + (ab+bc+cd+de+ea) = 0$, an equation of the second degree symmetrical in form.

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139. "Let e be the eccentricity, or ratio of polar compression to the equatorial radius of the earth. Find the average length of the earth's radius."

SOLUTION BY THE EDITOR.

THE solidity of an oblate spheroid $= \frac{4}{3}\pi a^2 b$, and its surface

$$= 2\pi a^2\left[1 + \frac{1-e^2}{2e}.\log\left(\frac{1+e}{1-e}\right)\right].$$

Let $\rho_1, \rho_2, \rho_3, \ldots . \rho_n$ represent the radius at n different points of the earth's surface, and let s represent one n^{th} part of the surface; then, if n is increased without limit, we shall have

$$s.\tfrac{1}{3}(\rho_1 + \rho_2 + \rho_3 + \ldots . + \rho_n) = \tfrac{4}{3}\pi a^2 b. \ldots \ldots . . (1)$$

But $(\rho_1 + \rho_2 + \rho_3 + \ldots . + \rho_n) \div n$ is the mean radius, and

$$ns = 2\pi a^2\left[1 + \frac{2-e^2}{2e}.\log\left(\frac{1+e}{1-e}\right)\right]. \ldots \ldots \ldots . (2)$$

By dividing the two members of (1) respectively by the corresponding members of (2) we get

$$\frac{1}{3}\left(\rho_1 + \rho_2 + \rho_3 + \ldots + \rho_n\right)\frac{1}{n} = \frac{2b}{3\left[1 + \frac{1-e^2}{2e}.\log\left(\frac{1+e}{1-e}\right)\right]},$$

or $$\left(\rho_1 + \rho_2 + \rho_3 + \ldots + \rho_n\right)\frac{1}{n} = \frac{2b}{\left[1 + \frac{1-e^2}{2e}.\log\left(\frac{1+e}{1-e}\right)\right]}$$

= the mean radius, nearly.

This solution is rigorously exact only on the supposition that the radii ρ_1, ρ_2, ρ_3, &c., are everywhere perpendicular to s, which is only the case, however, at the equator and poles. The maximum angle of the vertical, which occurs at latitude 45°, is, however, only about 11′, so that the solution gives a close approximation.

A solution of the question, not restricted to the earth, but that shall apply to any ellipsoid of revolution, can not be effected without resort to series, or elliptic functions, as the following solution will show.

SOLUTION BY ARTEMAS MARTIN, ERIE, PA.

LET a be the equatorial and b the polar radius of the earth, then $a^2-b^2 = a^2e^2$, and

$$b = a\sqrt{(1-e^2)}. \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\; (1)$$

Let R be any radius-vector drawn from the center of the earth to its surface; x and y the corresponding co-ordinates, the origin being at the center of the earth; S the surface of the earth, and $\varDelta$ the mean radius; then

$$R = \sqrt{(x^2+y^2)} \;.\;.\;.\;.\; (2), \quad \text{and } \varDelta = \frac{\int RdS}{\int dS} \;.\;.\;.\;.\;.\;.\; (3)$$

Assuming the earth to be an oblate spheroid of revolution, the equation to the generating ellipse is

$$a^2y^2 + b^2x^2 = a^2b^2, \;.\;.\;.\; (4), \quad \text{or, by (1), } y^2+(1-e^2)x^2 = a^2(1-e^2) \;.\;.\;.\; (5)$$

and
$$dS = 4\pi x\sqrt{(dx^2+dy^2)} = \frac{4\pi}{1-e^2}\sqrt{[a^2(1-e^2)^2+e^2y^2]}dy. \;.\;.\;.\;.\; (6)$$

Integrating (6) between the limits $y = a\sqrt{(1-e^2)}$ $(=au)$, $y = 0$,

$$S = 2\pi a^2+\frac{\pi a^2(1+e^2)}{e}.\log\left(\frac{1+e}{1-e}\right). \;.\;.\;.\;.\;.\;.\;.\;.\;.\; (7)$$

By substitution (3) becomes

$$\varDelta = \frac{4\pi}{S(1-e^2)^{\frac{3}{2}}}\int_0^{au}[a^2(1-e^2)-e^2y^2]^{\frac{1}{2}}[a^2(1-e^2)^2+e^2y^2]^{\frac{1}{2}}dy. \;.\;.\;.\;.\; (8)$$

Put $a^2(1-e^2)-e^2y^2 = w^2$, then $y = (1\div e)\sqrt{[a^2(1-e^2)-w^2]}$ and

$$\varDelta = \frac{4\pi}{Se(1-e^2)^{\frac{3}{2}}}\int_{au.u}^{au}\frac{\sqrt{[a^2(1-e^2)(2-e^2)-w^2]}w^2dw}{\sqrt{[a^2(1-e^2)-w^2]}}$$

Let $av\sqrt{(1-e^2)} = w$, and $c^2 = 1\div(2-e^2)$, then $dw = a(1-e^2)^{\frac{1}{2}}dv$ and

$$\varDelta = \frac{4\pi a^3(2-e^2)^{\frac{1}{2}}}{eS}\int_u^1\frac{\sqrt{(1-c^2v^2)}v^2dv}{\sqrt{(1-v^2)}},$$

$$= \frac{4\pi a^3}{ceS}\left[\int_u^1\frac{(1-c^2v^2)^{\frac{1}{2}}dv}{(1-v^2)^{\frac{1}{2}}} - \int_u^1\frac{[1-(1+c^2)v^2+c^2v^4]dv}{(1-v^2)^{\frac{1}{2}}(1-c^2v^2)^{\frac{1}{2}}}\right],$$

$$=\frac{4\pi a^3}{ceS}\left[\int_u^1\frac{(1-c^2v^2)^{\frac{1}{2}}dv}{(1-v^2)^{\frac{1}{3}}}-\frac{1}{3}\int_u^1\frac{[1-2(1+c^2)v^2+3c^2v^4]dv}{(1-v^2)^{\frac{1}{2}}(1-c^2v^2)^{\frac{1}{2}}}-\frac{1}{3}\int_u^1\frac{[2-(1+c^2)v^2]dv}{(1-v^2)^{\frac{1}{2}}(1-c^2v^2)^{\frac{1}{2}}}\right].$$

But $\int\frac{[2—(1+c^2)v^2]dv}{(1—v^2)^{½}(1—c^2v^2)^{½}}=(1+c^2)\int\frac{(1-c^2v^2)^{½}dv}{(1—v^2)^{½}}—(1-c^2)\int\frac{dv}{(1-v^2)^{½}(1-c^2v^2)^{½}}$,

and $$\int\frac{[1—2(1+c^2)v^2+3c^2v^4]dv}{(1—v^2)^{½}(1—c^2v^2)^{½}}=v\surd(1—v^2)\surd(1—c^2v^2);$$

$$\therefore\ \varDelta=\frac{4\pi a^3}{ceS}\Big[\tfrac{1}{3}(2—c^2)\int_u^1\frac{(1—c^2v^2)^{½}dv}{(1—v^2)^{⅓}}+\tfrac{1}{3}(1—c^2)\int_u^1\frac{dv}{(1—v^2)^{½}(1—c^2v^2)^{½}}$$
$$—\tfrac{1}{3}v\surd(1—v^2)\surd(1—c^2v^2)\Big],$$

$$=\frac{4\pi a^3}{3ceS}\Big[ce(1—e^2)^{½}+(2—c^2)[E'(c)]+(1—c^2)[F'(c)]\Big],$$

where F' and E' denote elliptic functions of the first and second orders.

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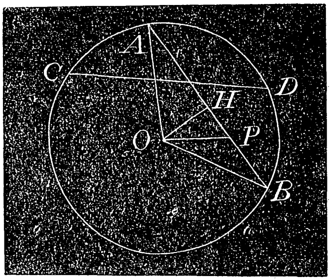
140. "A point is taken at random within a given circle, and a random chord drawn through it. If another chord be drawn at random, what is the probability that it will intersect the first?"

SOLUTION BY E. B. SEITZ, GREENVILLE, OHIO.

LET P be the random point, O the center of the circle, AB the random chord through P, and CD the second random chord. Draw OH perpendicular to AB. Put $OP=x$, $OA=1$, $\angle OPA=\theta$, $\angle AOH=\varphi$. Then we have $OH=x\sin\theta=\cos\varphi$, $x=\operatorname{cosec}\theta\cos\varphi$, $dx=—\operatorname{cosec}\theta\sin\varphi\,d\varphi$. The second random chord may be drawn from any point of the circumference to any other point: if it be drawn from any point of either of the arcs ACB, ADB to any point of the other, it will intersect the first. Hence while P and AB are fixed, the number of ways CD can be drawn to intersect AB is $2\,\text{arc}\ ACB\times\text{arc}\,ADB=8\varphi(\pi—\varphi)$; and the whole number of ways it can be drawn is $4\pi^2$. The limits of x are 0 and 1, of θ, 0 and $\frac{1}{2}\pi$.



Hence the required probability is

$$p=\frac{\int_0^{\frac{\pi}{2}}\int_0^1 8\varphi(\pi—\varphi)d\theta\times 2\pi x dx}{\int_0^{\frac{\pi}{2}}\int_0^1 4\pi^2 d\theta\times 2\pi x dx}$$

$$=\frac{8}{\pi^3}\int_0^{\frac{\pi}{2}}\int_0^1\varphi(\pi—\varphi)d\theta\,xdx$$

$$= \frac{8}{\pi^3}\int_0^{\frac{\pi}{2}}\int_{\frac{1}{2}\pi-\theta}^{\frac{1}{2}\pi} \varphi(\pi-\varphi)\operatorname{cosec}^2\theta d\theta \sin\varphi\cos\varphi d\varphi$$

$$= \frac{1}{\pi^3}\int_0^{\frac{\pi}{2}}(\pi^2 - 4\theta^2 + 2\theta^2\operatorname{cosec}^2\theta - 4\theta\cot\theta + 2)d\theta$$

$$= \frac{1}{3} + \frac{1}{\pi^2}$$

QUERY. "In Thompson and Tait's 'Elements of Natural Philosophy' it is stated that, 'at the southern base of a hemispherical hill radius a and density ρ, the true latitude (as measured by the aid of the plumb-line) is diminished by the attraction of the mountain by the angle $\frac{2}{3}\rho\pi a \div (G - \frac{4}{3}\rho a)$, where $G =$ the attraction of the earth in same units.' How is this proved?"

ANSWER BY HENRY HEATON.

As the angle is so small that it is practically equal to its tangent, if $G=$ attraction of the earth, $\frac{2}{3}\rho\pi a =$ horizontal, and $\frac{4}{3}\rho a =$ vertical attraction of the hill then it is evident that the angle of deflection of the plumb-line $= \frac{2}{3}\rho\pi a \div (G - \frac{4}{3}\rho a)$.

Where the unit of attraction is the attraction of a mass whose volume is 1, density 1, and distance 1, it has been shown, page 194, Vol. II, of the ANALYST, that the horizontal attraction of the hill $= \frac{2}{3}\rho\pi a$; and that the vertical attraction $= \frac{4}{3}\rho a$, I shall now proceed to show.

Taking the origin of co-ordinates at the center of the spherical surface, the attraction exerted by an element of the mass resolved in a vertical direction $= \frac{\rho dxdyzdz}{[(x+a)^2+y^2+z^2]^{\frac{3}{2}}}$. Hence the vertical attraction of the hill is

$$\rho\int_{-a}^{+a}\int_{-u}^{+u}\int_0^{v}\frac{dxdyzdz}{[(x+a)^2+y^2+z^2]^{\frac{3}{2}}} = \rho\int_{-a}^{+a}\int_{-u}^{+u}\left[\frac{dxdy}{[(x+a)^2+y^2]^{\frac{1}{2}}}\right.$$

$$\left.- \frac{dxdy}{[2a^2+2a]^{\frac{1}{2}}}\right] = \int_{-a}^{+a}\left[\log\left(\frac{\sqrt{(2a)}+\sqrt{(a-x)}}{\sqrt{(2a)}-\sqrt{(a-x)}}\right)dx - \frac{2\sqrt{(a-x)}dx}{\sqrt{(2a)}}\right]$$

$= \frac{4}{3}\rho a$; where the limits of integration, u and v, are, respectively, $u = \sqrt{(a^2 - x^2)}$ and $v = \sqrt{(a^2 - x^2 - y^2)}$.

If we put r for radius of the earth, and δ, its mean density, $G = \frac{4}{3}\delta r$. Substituting this for G in the expression for the angle of deflection, we get, $\rho\pi a \div 2(\delta\pi r - \rho a)$. As a is very small in comparison with r the expression for the deflection is, practically, $\rho a \div 2\delta r$.

[Mr. Adcock obtains precisely the same result as the last above written, viz.; $\rho a \div 2\delta r$.]